Temporal adaptation enhances efficient contrast gain control on natural images

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**Context**
The redundancy reduction hypothesis postulates that neural representations adapt to sensory input statistics such that their responses become as statistically independent as possible. Based on this hypothesis, many properties of early visual neurons—like orientation selectivity or divisive normalization—have been linked to natural image statistics. Divisive normalization in particular models a widely observed neural response property: The divisive inhibition of a single neuron by a pool of others. This mechanism has been shown to reduce the redundancy among neural responses to typical contrast dependencies in natural images. Here, we compare a standard model of divisive normalization to a functionally similar, but statistically optimal mechanism called radial facilitation.

**Mathematics**

**Redundancy — Multi-Information**

\[ I[Y] = D_{KL}(p(y) \prod_{i=1}^{n} p_i(y_i)) = - \sum \log p(y) - \sum \log p_i(y_i) \]

**Multi-Information Estimation**

\[ A(y|x) = - (\log p_i(y_i)|y=y(x)) = H(Y) - D_{KL}(p(y|x)\| p(y)) \]

\[ I[Y] = \sum \log p(Y) - \log p(Y|X) \]

**Naka-Rushton Distribution**

\[ \nu \sim (r_{A}, r_{B}, s) \Rightarrow r_{N-R} = \frac{a r}{\sqrt{a^2 + r^2}} = \frac{\nu}{\sqrt{\nu^2 + 1}} \]

\[ \nu(r_{A}, r_{B}, s) = \frac{2 a^2 s^2 \exp(-\frac{a^2}{\nu^2 + 1})}{(\frac{2 a^2}{\nu^2 + 1} + 1) (\frac{2 a^2}{\nu^2 + 1} + 1)} \]

**Summary**

- **Divisive normalization** has been shown to reduce higher order dependencies on natural images.
- **Radial facilitation** is an optimal redundancy reduction transform for these two mechanisms.

**Dynamic Model**

A: Histogram of \( |y| \) for natural image patches sampled with simulated eye movements: The dynamically adapting \( \sigma^2 \) predicts a mixture of Naka-Rushton distributions for \( |y| \), which closely matches the empirical distribution.

B: Redundancy reduction performance for simulated eye movement data. The dynamically adapting \( \sigma^2 \) achieves an almost optimal redundancy reduction performance. C: Dynamics of the adaptive \( \sigma \): values of \( r_{A} = |y| \) plotted against \( \sigma_{A} \), adapted to \( r_{A-1} \). The correlated values indicate that the shift of the contrast response curve (controlled by \( \sigma \)) tracks the ambient contrast level.

**Further Information**

Code and papers are available at [http://www.bethgelab.org](http://www.bethgelab.org)